DOCUMENT RESUME

ED 155 068 SE 024 410

AUTHOR Robinson, Floyd G.
TITLE Problem Solving: Creating a Trend Toward a Shared Image.
PUB DATE Mar 78
NOTE 37p.; Paper presented at the annual meeting of the American Educational Research Asscciation (Toronto, Canada, March 27-31, 1978)

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage.
DESCRIPTORS *Curriculum Development; *Mathematics Education; *Problem Solving; Productive Thinking; Program Development; *Scientific Methodology; *Teaching Skills

ABSTRACT

This paper outlines the increasing prospects for improving the problem solving component of future mathematics programs. Of particular interest is the development of a viable growth schema for general problem solving strategies and skills. The plan presented here provides for five levels of problem solving sophistication. The main body of the paper is devoted to the description of these levels. Evidence drawn from two projects is used to assess the feasibility of movement from level to level, and to comment on requirements for, and obstacles to, that movement. (BB)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Howard. Russell for Floyd Robinson

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) AND USERS OF THE ERIC SYSTEM."

ED155068

PROBLEM SOLVING: CREATING A TREND TOWARD A SHARED IMAGE

Paper presented to the Annual Meeting of the
American Educational Research Association
Toronto, Ontario
March 27-31, 1978

by

Floyd G. Robinson

The Ontario Institute for Studies in Education
Northeastern Centre

SE 024 410

Abstract

Today's teacher of mathematics, in common with colleagues in other disciplines, finds himself pulled by two apparently divergent forces. On the one side, the professional literature is saturated with the image of the "real life problem solver", an individual who is intelligently self-directed in the significant contexts of his everyday life. On the other side, a narrowly conceived "back to the basics" movement, backed frequently by the authority to impose its own evaluation standards, exerts pressure on the teacher to limit his efforts to a narrow band of algorithmic competencies.

A first contention of this paper is that the "back to the basics" movement does not reflect a different image of the educated person, but rather (i) a lesser capability for tolerating the frustration of not moving discernibly closer to the shared image, coupled with (ii) an inability to analyze out the skill components of real life problem solving. The real danger to mathematics education, it is contended, will come when the profession itself abandons its liberal images. And the real antidote to a narrowly conceived "back to the basics" movement is to implement the real life problem solver image to some discernible degree, rather than merely urging its merit.

The now substantial theoretical and research literature on implementation offers some clues as to how we might proceed. In brief, it is proposed that we should conceptualize the implementation of our professional image in terms of supporting teacher behaviors which are arranged in some kind of levels or phases of growth, the most sophisticated being those needed to sustain the image in practice, and that we should address seriously the task of moving the teacher from level to level.

The main body of the paper is devoted to the description of such a proposed set of phases. Evidence drawn from two major OISE projects (The Major Thrust in Elementary School Thinking and Curriculum Decision Making Project) is used to assess the feasibility of movement from phase to phase, and to comment on requirements for, and obstacles to, that movement.

The conclusion is optimistic: there is no warrant for abandoning our image of the real life problem solver. Incorporating what we now know and are able to do in teacher pre-service, in-service, and graduate programs would produce a product that the "back to the basics" exponent would recognize as being much more congruent with the shared image than he is himself.

PROBLEM SOLVING: CREATING A TREND TOWARD A SHARED IMAGE

Over the past decade there appears to have been a strong resurgence of interest in problem solving among mathematics educators. This interest has been reflected in the formation of special interest groups in mathematics problem solving,[1] in special editions of professional magazines and in the high ranking of problem solving in current lists of "basic" skills.[2]

The fact that similar claims could be made for increasing interest in complex intellectual performance in most other school disciplines leads one to suspect that the resurgence of concern for problem solving performance is but the surface feature of a more fundamental change in the shared image of what it means to be educated. A widely shared image that appears to have crystallized in the early 1970's is of the "real life problem solver", an educant who is intelligently self-directed in the various contexts of his lived experience: in his dealings with nature; in the family; at productive work; at leisure; and in the broader arenas of social and political action. This image has not only spawned entire new school programs--values clarification, decision-making, moral reasoning, family studies--but has generated a vigorous "real life problem solving" emphasis in most of the established disciplines in the secondary school. Thus in the recently issued Ministry of Education guidelines for the intermediate grades (7-10 in Ontario), we find a greatly enlarged emphasis on: the analysis of contemporary issues in history; man as chooser in geography; problems of environmental abuse and energy use in science; and the practical skills of written and spoken communication in English.

In mathematics, the real life problem solver has most commonly been envisaged as a person who builds models of problematic situations, manipulates them symbolically, and projects the implications of the result back on the phenomena he is engaged with. This view is strongly urged in current Ministry directives to teachers in both the K-6 and intermediate panels. In fact, in the former case, the model building view was raised to the status of an epistological decree when the teacher was informed that:

> Fundamentally, (mathematics) consists of abstract models that represent aspects of the real world, and with rules and operations for working with these models....The process of mathematics consists of choosing or creating models; of studying, analyzing, and manipulating them; and of relating the findings back to the real world....These models help children to see relationships in the environment and to make sense of the world about them.[3]

While Ontario may have idiosyncratic excesses, Babin et al.'s (1975) extensive survey of trends in contemporary intermediate level mathematics programs elsewhere suggests that our current guidelines merely reflect a conception that is very widely held.[4]

Threat of the "Back to the Basics" Movement

Of course the mathematics education literature now contains another dominant theme: the so-called "back to the basics" movement. Those who are heading this movement have used their considerable political power to propose or establish province- or state-wide testing programs. Almost invariably they have opted for a very narrow range of algorithmic skills and related knowledge. Spokesmen for the teachers' associations are currently voicing their apprehension, the principal argument being that the emphasis in these testing programs is antithetical to our (presumably shared) image of the mathematically educated person.

It is the author's contention that neither the political spokesmen for the movement, nor the people they represent, have a view of the educated person that is significantly different from that advocated in the professional literature. The man on the street has been incessantly bombarded with the image of the intelligently self-directed person--it, or its antithesis, is the mainstay of popular fiction and television--to the point that he has internalized it as a dominant value of our contemporary life style. Evidence of this comes from the virtually countless studies of the aims of education conducted in this province in the past decade; when asked to formulate or choose among objectives for the school, laymen invariably put at the top of the list aspects of the ability to deal with the practical problems of life from a consistent set of

values; the so-called "basic skills" appearing in current "back to the basics" achievement tests in mathematics have tended to receive honourable mention at best.

Public support for the "back to the basics" movement does not derive, then, from a different conception of what it means to be educated. Rather, it stems from the layman's inability to sustain confidence in this image in the face of frustration and short term failure. The layman, whether he be citizen or employer, has been given very little data over the past decade to indicate growth in intelligent self-direction on the part of the school's graduates. What he has seen is a destructive and short-lived rebellion against tradition and the establishment, followed by data which show that performance in some of the so-called "basic skills" is declining. Unable to sustain his belief without positive external reinforcement, and unable to analyze out the achievable and educable components of the shared image, the layman quite naturally focusses his attention on the most easily observed indices of performance, and argues that at least these deficiencies should be overcome. There is nothing unusual in this phenomenon: most of us in our youth had images of what we might become or accomplish, but in the face of failure we progressively become frustrated and seek out more immediate and achievable indices of success, even though we know these are but tangentially related to our ideals. The "I may not have achieved what I wanted to, but I am at least making good money and living comfortably" formulation encapsulates the lot of the vast majority.

It is an easy and perhaps inevitable next step to rationalize the substituted criteria and to imagine that they indeed have something to do with the ideal. Reference to narrowly conceived testing objectives as "basic skills" or "life skills" or even "survival skills" shows how far this process has already proceeded. No plausible analysis or data has been presented to show how the outcomes demanded in these lists contribute to the behaviors to be exhibited by the intelligently self-directed person.

Thinking on this matter is barely above the level of simple association. For example, a highly placed objective on every list of basic mathematical skills is the ability to compute simple percent, the argument being that real

life money transactions involve interest. Yet the most important financial transaction that most laymen engage in has to do with taking on a mortgage, and understanding simple interest will not go very far in helping him understand the complexities of compound interest, late penalties, forfeit clauses, and the like. And it would contribute even less to the individual's capacity to make a reasoned decision as to the size of the mortgage he can cope with, the latter involving such complexities as anticipated earning patterns, competing demands for income, and a host of other factors. The root of the layman's problems is that he cannot construct a framework for the problematic situation that is sufficiently comprehensive to encompass the many elements involved: once he has such a framework, skill in simple percentage calculations would certainly help, but it is a mistake to believe that it would in itself contribute much to intelligent self-direction in financial affairs.

One might confidently predict that the "back to the basics" movement will itself encounter public disenchantment when it is discovered--as experience has taught us many times--that the development of a high level of proficiency in algorithmic skills carries no guarantee of intelligent action in the real world. The student who can compute efficiently, but who shows no growth in intelligent self-direction will, in time, come to rasp the public nerve even more than the graduates of today's schools. Then we will jettison the tests, at least as they are presently conceived, and a new cycle will begin. The danger, of course, is that under pressure many teachers may themselves temporarily abandon their professional ideals, taking on a "if that's what they want, that's what they'll get" posture, which could cause mathematics education to take a long step backward.

It is all to the good that mathematics teachers' associations are reiterating their faith in the conception of the real life problem solver. While such reaffirmations should continue, we must also take definite steps to <u>implement</u> this image, to bring it into practice. In the long run, this is the only sure route to its maintenance and progressive development.

Implementing the Shared Image

From the extensive analysis of educational change that has been undertaken in the past few years, we have learned how rarely the aspirations and enthusiasms of would-be curriculum improvers have actually led to substantial changes in classrooms and student performance, even when these innovations are backed by extensive programs of applied research and materials development. This failure has prompted attempts to reconceptualize both the process of planned change in general[5] and of innovations in particular.[6,7]

Two significant insights come from this literature. First, innovations may be most productively conceived in terms of the categories or dimensions of teacher behavior that are required to support the innovation.[8] Second, implementation is not a "one-shot" process, but proceeds by phases or stages; these stages can be defined by describable changes in teacher behavior along each of the proposed dimensions of change.[9]

These developments lead the author to believe that we can only have grounds for optimism that something of significance will come out of our renewed interest in problem solving if we traverse the gap between reality and image with a set of discreet steps--changes in teacher behavior--that are deemed feasible at our present stage of knowledge and commitment, and if we address ourselves very directly to the task of making these changes occur.

In the accompanying table, the author sketches a sequence of such steps. While a large number of potential dimensions of teacher behavior are available from the classical and contemporary curriculum literature, the particular choice of both dimensions, and levels within the dimensions, was considered most useful in allowing the author to synthesize and articulate his experiences in two major projects of The Ontario Institute for Studies in Education, undertaken over the past eight years, which have dealt with various aspects of intellectual performance. In The Major Thrust in Elementary School Thinking,[10] a group of OISE staff members developed and tried out in classrooms a large quantity of materials intended to stimulate a range of thinking activities.[11] This was followed by an attempt to extract the procedures by which these materials were

developed[12] and a currently operating experimental program in curriculum consultancy which instructs elementary and secondary teachers in the use of these procedures to construct new curriculum materials.[13]

In the sections that follow, the main characteristics of each stage, and the conclusions we have drawn concerning the feasibility of producing the type of growth indicated, will be commented upon.

Level 1: unremitting focus on algorithmic competence: At this level the teacher views a "problem" as a difficult application of an algorithm, and mathematics instruction is programmed almost entirely on the basis of increasing algorithmic sophistication. Problems are chosen or concocted to illustrate the level of algorithm under development or consolidation, and their solutions predictably call for the use of that type and level of algorithm. The student's ability to use algorithms efficiently and accurately is the teacher's main criterion for the success of his program, and both testing and remedial teaching focus on such performance.

Our evidence indicates that this is still the predominant orientation to mathematics instruction in Ontario schools today. This evidence comes from the analysis of textbooks in use, and from data showing the teacher's heavy dependence on textbooks programmed primarily for increasing algorithmic proficiency beyond the primary grades[14] and is complemented by the observations of classroom teachers themselves.[15]

The consequences for problem solving performance of this orientation to mathematics instruction is not difficult to anticipate. The time-honoured complaint that the majority of mathematics students have great difficulty with problems not specifically rehearsed in class, and identified with such rehearsals on test situations, is abundantly clear in current data on problem solving performance in Ontario's elementary and secondary schools.[16,17] And when non-textbook type questions of a logical or quantitative nature are used, the results are even worse, with the majority of students scoring only marginally above chance level.[18]

Perhaps even more allarming than performance data is evidence that, very early in their school careers, students come to believe that the mathematics they study in school has very little to do with their interests outside school. In the Russell et al. study,[19] only a small minority of students were able to propose how they might use the computational algorithms they were rehearsing at the moment in the activities in which they engaged outside school. For the remainder, the purpose of the exercise was obvious, "It's in the book", or "The teacher makes us do it" being the most common answers to the question, "Why do you think they want you to know how to do this?" While we would not contend that elementary school children can be expected to be fully aware of the uses of school-based mathematics they do make in their daily lives, the early dissociation of mathematics from real life is certainly at far remove from the prevailing image of the mathematically educated person.

Level 2: uncoordinated use of idiosyncratic problem solving models: At this level, the teacher, now more experienced, conceives of problem solving in terms of a simple model, in which computation constitutes one of several skills. The possession of such a model prompts the teacher to seize opportunities to strengthen components other than algorithmic performance, such as formulating a question, drawing a diagram, or estimating or checking out the reasonableness of answers, but the teacher has little notion of how these components grow beyond the level at which she is currently working.[20] Increasing algorithmic proficiency continues to provide the main basis for programming, focussed teaching, evaluation, and remedial action.

In our experience it is not difficult to have teachers articulate such a model by reflection on how they solve problems normally encountered by their students. Most of the models that emerge from such an exercise are variants of the ones found in the professional mathematics education literature: Forbes,[21] for example, proposes that "the ability to solve real world mathematics problems involves the ability to accomplish the following: formulate the problem; recognize that mathematics is relevant to its solution; select the

appropriate mathematical objects and operations to use; carry out the computations accurately and efficiently; and interpret the computational results that produced the solution to the problem".

Evidence that training in the components of problem solving other than algorithmic proficiency can yield gains in problem solving performance can be found in several of the studies appearing in The Journal of Research in Mathematics Education, and we have attained similar results in the pilot tests of materials developed in The Major Thrust in Elementary School Thinking. As an instance of the latter, it was found that giving the students a concrete representation of the elements of a rate and ratio problem yielded significant improvement in performance across a broad range of problems involving rate and ratio, percentages, commissions, interest, and the like.[22]

The movement of teachers from level 1 to level 2 would be a useful, though modest, step forward in the author's opinion. At the very least, the model would force teachers to reflect that problem solving competence does not follow automatically from algorithmic proficiency, and to begin to look for opportunities to develop other skill components. It is to be doubted whether such disjointed efforts would in themselves have much cumulative effect; for this to occur, we must turn to the next level.

Level 3: component skill development within a shared conception of problem solving: At this level, the problem solving training of a student over a period of years (e.g., a school division) would be envisaged in terms of a set of skills (and accompanying model) to which a group of teachers jointly subscribe. Although a textbook programmed on the increasing sophistication of algorithmic skill may remain the chief organizational device for instruction, teachers would systematically include special units to develop other components of problem solving, and such units would demand increasingly sophisticated use of these other skill components. Special tests, or parts of regular tests, would be given to determine progress in such components, and some remedial action would be undertaken when such progress was not in evidence.

Again, it has been our experience that it is possible to have a group of mathematics teachers generate such a model (set of component skills) by reflecting on the range of problems they intend to deal with in their division, the result usually being a more elaborate model than that proposed by an individual teacher, and containing at least the beginnings of such real life problem solving requirements as: formulating the question; obtaining relevant data; and judging whether or not the derived answer makes sense. At least one school district in Ontario has developed model-based special units for the development of non-algorithmic skills and has scheduled these over the junior and intermediate elementary grades.[23] Similarly, a university-based group has been working with engineering students in problem solving tutorials; guided again by a problem solving "strategy" the group hopes to strengthen component skills systematically over the four years of the student's engineering training, stressing in their initial work the organization of the lecture-based knowledge required to solve problem assignments.[24]

There is also evidence to suggest that making the model which incorporates the skills available to the student, hence raising problem solving to a reflective level, helps to organize and improve problem solving performance. The Major Thrust in Elementary School Thinking group had considerable success in enhancing problem solving performance by teaching a related set of inquiry models for cause-effect analysis, case studies, and decision-making.[25] The same idea has been applied with success to correlational analysis[26] and critical reading.[27] One major advantage of the approach is that it allows both the teacher and student to focus precisely on the aspect or component of the problem on which the student is stuck; in other words, it allows a much more precise shared discussion of the process of problem solving.

Level 4: growth schemes for component strands within a shared model: The difference between this level and the previous one would be that the non-algorithmic components of problem solving would be programmed by some consistent principle, and over a broader time span, so that a student's performance could be described in terms of a profile of his performance on component skills.

When a school system reaches this point, each of the hypothesized component skills of problem solving would have the same status as algorithmic proficiency in terms of teaching, evaluation, and remedial action. A corollary would be that the student's total mathematics problem solving performance would constitute the main basis for his placement or "promotion" from level to level.

While no group the author knows of has proceeded quite this far, the claim can be made that the ingredients for such an undertaking are available. For one thing, the appearance of Gagné's task analysis was followed by many useful analyses of skill hierarchies. In The Major Thrust in Elementary School Thinking, we used a combination of Gagné-type analysis and observation of students to devise the skill sequences that are contained in the extensive materials emerging from that project. More recently, we have been teaching a procedure for devising skill sequences (or for devising a variant of sequenced objectives that we call a "growth scheme") and have found that teachers who use these procedures improve their products, both in respect to the precision of the behavioral statements and overall coherence, above sequences produced by the typical method of group speculation on possible stages of growth.

Two major gains would be achieved at this level, the first being to force awareness of evidence of enormous gaps in our mathematics programs when they are examined from a levels-of-component-skills perspective. One type of gap has to do with completeness within a skill category. For example, most teachers appear to believe that drawing diagrams (making a representation of the problem situation) is a useful component skill, but few realize how limited a set of representations we actually assist students to develop. Even the primary grade child lives in a world in which ratio (uneven dividing up), directed distances (finding places), classifying in all its variants, combinations (choosing small teams), coordinated time lines (the basis of everyday planning), optimal paths (cleaning the yard) and even simple input-output systems (the transactions at a piggy bank) are all elements of genuine problematic situations, yet in school he is limited for the most part to diagrams that represent the four "basic" operations on aggregates.

A second type of gap emerges when we compare the detail in our component skill programming with what we do in respect to algorithmic competency. The treatment of classification provides a case in point: despite the acknowledged importance of classification (many people consider it the primary basis of organized thought), the typical Ontario student's exposure comprises: physical (one-dimensional) sorting in the primary grades (usually with sets of numbers); a bit of Venn diagramming in the senior elementary grades; an exposure to the notion of "keys" in grade 7 and 8 science (not linked in the student's mind with Venn diagrams), and exposure to the notion of assumptions and deductive proof somewhere in the high school mathematics program. No effort is made to progressively expand classification skills from their primitive physical basis, to show the equivalence of classification systems used in different school subjects (e.g., Venn diagrams, numerical cross-classification tables, tree diagrams) or to develop the conception of logical thought and proof as minor extensions of the student's earlier classification work, although all these clearly can be done.[28]

A second gain from a levels-of-component-skills analysis is that the precise definition of a skill level allows a precise articulation of the "know how" or procedure underlying the skill. Skill teaching is almost always done by modelling, and where the skill has been analyzed and "levelled", the teacher's demonstration is normally accompanied by a running verbal statement of the procedure he is executing. But for many if not for most of the component skills of problem solving, the teacher merely models the behavior, in many cases because he has not himself articulated the procedure he uses (e.g., how do we select an appropriate representation of the problem situation? Or, how does the skilled problem solver go about retrieving from memory the stored knowledge that is most likely to bear on the problem at hand?). In the author's opinion, much more articulation of the procedures underlying components of problem solving skills should be attempted, and having a precise level of skill is an essential first step toward such articulation.

Level 5: interdisciplinary component skills analysis approach to problem solving: At this level, teachers in different subject disciplines would see themselves as developing and utilizing a common set of intellectual skills and as cooperatively enlarging the student's problem solving capabilities across a broad range of in-school and out-of-school tasks. Teachers of a particular discipline would be aware of the variants--in name and form--of skills employed in adjacent disciplines, the level of skills required at a particular stage in those disciplines, and the particular responsibility of his own discipline for developing the skill at the level required. The student would view his own efforts and progress in terms of using this common set of skills in varying knowledge contexts. Some measure of uniformity of growth across such contexts would be expected, and failure to progress in certain of these contexts would signal a need for remedial teaching (e.g., the student who had a highly developed ability in science inquiry, but poorly developed skill in analyzing social issues to which the products of his inquiry might be applied). At this level, much of the content for problem solving would be drawn from contemporary issues and concerns and become the object of interdisciplinary study in which the particular skill variants and knowledge of the various disciplines are brought to bear in a systematic way.

Obviously, enormous and probably insurmountable obstacles stand in the way of full attainment of this stage, not the least of which is the finite human capacity for adaptiveness and acquiring new knowledge. However, our current work with high school subject specialists in the experimental M.Ed. program in curriculum consultancy suggests that at least a modest and useful beginning can be made. The approach we have used is to have each discipline specialist in turn identify particular types of problems he expects a mature student to be able to solve, to reflect on how he (the teacher) solves them himself, and to draw from this analysis a list of skills he believes to be operative in mature behavior. Once a number of such lists have been developed, commonalities are encapsulated into a more general skills language, and lists for a particular discipline are extended as gaps are discovered by comparison.

The final product of such an exercise usually produces a list containing at least the following component skills:

establishing a focus for the inquiry: (which can take such forms as posing a problem question, stating an hypothesis, or identifying a decision to be made);

establishing a framework for the inquiry: (which can take such forms as drawing a diagram of the problem elements; establishing the structure of a table that must be completed to answer the question; drawing a sketch of the conceptual relationship between ideas to be expounded in a composition);

determining sources of data: (which has to do essentially with knowledge of data locations);

obtaining/interpreting data at the source: (which can take such forms as measuring physical dimensions; surveying opinion or practice; retrieving information from such external stores as books, maps, or artifacts; retrieving information from memory);

assessing the adequacy of data at the source: (which involves such things as determining accuracy, reliability and relevance, assessing bias; determining the (epistemological) status of data);

projecting data into the problem frame: (which can take such forms as putting numbers in tables; deducing unknown parts by application of propositional knowledge; filling in the details of a composition sketch);

reducing data to summary form by means of a data processing algorithm: (which can take such forms as determining preponderance or "weight", averaging, or solving a system of equations of various degrees of complexity);

observing relationships in summary data (which can take such forms as determining trends, comparing magnitudes);

interpreting the summary data: (which can take such forms as concluding that the summary symbols are the solution to the problem; ascertaining that a causal relationship is in evidence; declaring that a particular hypothesis is most plausible in light of the evidence; and asserting that a thesis or hypothesis has been supported);

extrapolating the interpretation beyond the problem framework: (which can take such forms as generalizing, predicting or suggesting the application of a particular line of reasoning to a new issue);

communicating the inquiry and its result: (which can take the form of demonstrating a proof; writing up an experiment; explaining how a particular alternative was deemed to be most appropriate; formally writing out an essay).

Though too general in nature to provide the basis of programming in a particular discipline, such a scheme provides an organizing conception for an interdisciplinary staff's discussion of their shared responsibilities for complex intellectual performance, and for the alignment and meshing of their objectives for maximum effectiveness. It would also provide a framework for the student, by which he could see continuity and relatedness in his problem solving efforts in different disciplines, and who would in time, and with continued practice, use this framework in a highly reflective yet habitual manner to assess the sophistication of his approach to problems in both school and real life.

From this start, a school staff can move on to the next stage, which would be to define commensurate levels of growth in the particular skill variants in use in the various disciplines. A particularly useful starting point in our experience is the skill of developing (what we term) the framework for the problematic situation. As an example, one might expect to find, in a coherent interdisciplinary approach to problem solving, the progressive development of the student's ability to organize data in tabular form, whether the student be working in mathematics, geography, economics, history, or whatever. Moreover, one can define at least a rough growth scheme for the student's ability to

establish such frameworks by observing that growth in this respect is reflected in the use of an increasing number of dimensions, in the complexity of the concepts used to define the dimensions, and in the strength of the scale used to relate subdimension values. Thus in studying the relationship between community size and types of work, the primary grade child might be content to tabulate the number of workers of a particular kind that he knows (doctors, policemen) in two communities familiar to him, while a much more sophisticated student of geography might want to find out how the relationship between community size and distribution of population by occupational types is influenced by the distance of communities from large metropolitan centres (the answer to which would constitute a modest contribution to central place theory). While these two inquiries differ in respect to the maturity required in all the skills identified earlier, our main interest here is that the child's framework is one-dimensional with low order defining concepts linked by a nominal scale, while the advanced student's is three-dimensional, whose dimensions are defined by much more sophisticated concepts, and whose subdimensions are linked by ordinal and ratio scales respectively. Obviously there is room for finely detailed programming between these two end points, using the increasing values of the three variables mentioned as indices of growth.

The particular problem solving concerns of the various high school subject disciplines are reflected in large measure in the types of factual, conceptual, and propositional knowledge they project into such frameworks. Since there is a substantial overlap in the content of related disciplines, the inter-disciplinary analysis of frameworks for common problems provides a useful starting point for sorting out conceptual commonalities and variants. Since the framework also provides the primary organizing structure for written or oral communication of the results of a wide variety of inquiries,[29,30] teachers of English composition can find a common ground for substantive interaction and joint planning with subject specialists--a collaboration much applauded but rarely appearing in practice.

Reference to the inextricable link between skill and knowledge should alert us to beware of simplistic solutions to the immensely difficult tasks

presented by level 5 as it is described here. Many popular books and articles have outlined general problem solving models which are alleged to have general applicability across disciplines. Some of these models--particularly general models for decision-making--are quite useful in providing coherence to the problem solving efforts of unsophisticated students (who lack precise substantive knowledge to any marked degree) or for unsophisticated problematic situations. But in real life, as in the academic disciplines, knowledge and skill are reciprocally limiting entities, and a skill program cannot be expected to accomplish much if it does not come to grips with the organization and growth of academic (i.e., precise) knowledge.

Level 6: approaching the shared image: At this level, problem solving would be conceptualized in terms of a general set of intellectual skills whose discipline-specific variants establish, fill, and execute the frameworks for problematic situations in a representative array of real life contexts. Consequently, significant "developmental" problem solving tasks, as they are perceived in these contexts by students, would constitute a major programming vehicle, and the school's primary concern would be adequate performance in these contexts.

Teaching would emphasize unique qualities of real life problem solving (such as the need for "optimal" rather than the "right" solutions). One of the genuinely significant differences between real life and school problems is that the framework for the former can be developed to whatever level of complexity one chooses. An individual's economic analysis in buying a car can be limited to the comparative pricing of a particular kind of Buick (the focus for the inquiry being, "Where can I buy the cheapest Buick?"). On the other hand, one could create a massively complex framework for the same problematic situation (the orienting question being, "What car purchase would be the most economical, given existing performance data and projections of my current driving patterns?"); and one could proceed to make the problem even more complex by assigning dollar valuates to such factors as safety and comfort. Part of the task of preparing the student for real life problem

solving is to engender some sense of what problematic situations can be profitably analyzed, and how far this analysis should proceed in view of the value of the perceived outcome. When an appropriate level of the framework complexity is chosen for a problem, the real life problem solver will find additional difficulties in that he may have access only to incomplete or inadequate data, that he must frequently reach a solution very rapidly and without the aid of computational devices, that real life problem solving must frequently be undertaken in circumstances that are distracting or disruptive, and that some types of problem solving involve an interplay between two parties (as when husband and wife debate how much of the family funds should be expended on a purchase) in which the data and focus of the problem are constantly shifting. In all these and other respects, real life problem solving is different from the much tighter exemplars studied in school situations, but there is no reason why some of these differences could not become the object of systematic study in school situations. In short, the transition from the levels 5 to 6 should not be an insurmountable one; the real difficulty rests in reaching level 5.

Summary

This paper, though admittedly speculative in respect to levels 5 and 6, reflects the author's experience over the past decade in the collaborative development of skill oriented curriculum materials and in integrated on-site graduate instruction in development and implementation strategies. The conclusion we have drawn is that the technologies of skill analysis, methodology, and implementation are now sufficiently advanced to bring us to something akin to our level 4, and with a continuing thrust in knowledge programming, to level 5.[31]

Moreover, our experience in The Major Thrust in Elementary School Thinking has shown that it is not difficult to restore the layman's confidence that the shared ideal is at least partly attainable. When we began our inquiry training

program in the early 1970's,[32] we encountered many skeptical parents and teachers, and found that the best antidote was to have them watch inquiry-trained students deal with problems involving "real life" context. Although the curriculum developers (who also acted as demonstration teachers) were themselves operating at (a marginal) level 3, adults were consistently impressed with the students' performance, possibly because their own (adult) level of problem solving performance was not really that high. With the tests we have used, our data long ago convinced us that even a modest improvement in general problem solving ability would bring the intermediate grade student beyond the typical non-specialist adult level.[33]

The major obstacle to the implementation of the shared image is lack of time, particularly the most precious of times--that available to the teacher to upgrade his own competencies. The notion that significant improvement in our school programs would take place if curriculum technology were employed to produce "teacher proof" materials has not fared very well in practice; the current implementation literature is virtually universal in asserting that a curriculum innovation produces change only to that degree that teachers internalize the values inherent in the innovation and develop the knowledge and skill required to execute its objectives.

Insofar as programs intended to enhance complex intellectual performance are concerned, our teachers (Canadian) get a very shaky start in their pre-service training toward developing competencies that the execution of such programs require. In-service education in this country, and the associated professional literature, seem stalled at level 2: virtually endless "one-shot" exposures to suggestions for teaching specific topics or developing particular skills; there is little in it that would engender cumulative growth, or comprehensive planning by a staff or curriculum group of the type envisaged at level 3.

While some modification of both in-service and pre-service education may be expected over time, graduate level instruction focussed on curriculum development and implementation, and coordinated with ongoing local curriculum

work, appears from our experience to be the most promising way of effecting the sorts of changes in teacher behavior envisaged in levels 3, 4, or 5. Some of our better M.Ed. graduates are currently doing quite acceptable level 4 work with curriculum committees and school staffs, and one promising level 5 project is now underway on a small scale. We will have taken a giant step toward making real life problem solving a reality when every school staff contains some comparably skilled person.

References

1. Babin, P., Pogue, P., and Knoop, R. Recommendations For Intermediate Division Mathematics in Ontario, Based Upon a Study of Local and International Programs, University of Ottawa, 1975.

2. See, for example, statements by the National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics in The Mathematics Teacher, Vol. 71, No. 2, February, 1978.

3. Ontario Ministry of Education, Education in the Primary and Junior Division, 1975, p. 61.

4. Babin, et al., op. cit.

5. Leithwood, K., Russell, H., Clipsham, J., and Robinson, F. School Change: Stages, Constructs and Research Methodology. Interchange, Vol. 5, No. 4, 1974.

6. George, A., Rutherford, W., Hall, G., and Loucks, S. Reports of Recent Research on the Implementation of Innovations in Schools and Colleges Using Key Dimensions of the Concerns-Based Adoption Model. A series of papers presented to the Annual Meeting of the American Educational Research Association, San Francisco, 1976.

7. Fullan, M. Research on Curriculum and Instruction Implementation. Review of Educational Research, Vol. 47, No. 2, pp. 335-397, Spring 1977.

8. Fullan, M., op. cit.

9. George, et al., op. cit.

10. Robinson, F. G. The Major Thrust Project in Elementary School Thinking. Orbit, November, 1976.

11. These are described in Lamkow, H. Catalogue of the Thinking Materials Resource Centre, Niagara Centre, OISE, St. Catharines, Ontario, 1976.

12. Robinson, F., Clipsham, J., Hedges, H., and Popp, L. ICPOGMU: A System of Curriculum Development. Notes for OCA Conference, June 1977.

13. Clipsham, J. The M.Ed. Program in Curriculum Consultancy. Department of Curriculum, OISE, 1976, mimeo.

14. Russell, H., Robinson, F., Wolfe, C., and Dimond, C. Current Ontario Elementary School Mathematics Programs. Toronto: Ontario Ministry of Education, 1975.

15. Munn, A. Student Errors in the MAT: Their Relationship to a Lack of Problem-Solving Ability and Suggestions for Remedying This. OISE, 1977, mimeo.

16. Hedges, H. Achievement in Basic Skills. Toronto: Ontario Ministry of Education, 1977.

17. Munn, A., op. cit.

18. These data are not published. The author designed, and analyzed the results from, a problem solving test which formed part of a grade 8 test battery administered in some 30 school boards in Ontario in 1977.

19. Russell, et al., op. cit.

20. In the Russell, et al. study, teachers were given a set of skills identified by a province-wide committee of mathematics consultants as operative in real life problem solving situations, and were asked to give examples of the expected levels of student performance in these skills at the grade level they were presently teaching, and at grade levels below and above this. Only an insignificant proportion of teachers could give the barest outline of growth in relation to such skills.

21. Forbes, J. "Some Thoughts on 'Minimal Competencies'", The Mathematics Teacher, Vol. 71, No. 2, pp. 94-100.

22. A summary of evaluative studies is found in Zamkow H. Catalogue of the Thinking Materials Resource Centre, Niagara Centre, OISE, St. Catharines, Ontario, 1976.

23. Peterborough County Board of Education. Problem Solving to Accompany a Continuum of Basic Component Skills (Grades 4-6 and Grades 7-8), Peterborough, Ontario, 1974, 1975.

24. Woods, D., Wright, J., Hoffman, T., Swartman, R., and Doig, I. "Teaching Problem Solving Skills", Annals of Engineering Education, Vol. 1, No. 1, 1975.

25. Robinson, F., Brison, D., and Tickle, J. Inquiry Training: Fusing Theory and Practice, Profiles in Practical Education No. 4. Toronto: OISE, 1972.

26. Millard, T. African Climates, Soils and Vegetation: A Unit for Senior Elementary Schools Developed by Means of Correlational Analysis, The Thinking Materials Resource Centre, Niagara Centre, OISE, St. Catharines, Ontario, 1974, mimeo.

27. Boire, C. Critical Reading. Paper presented to the Annual Conference of the Ontario Council for Educational Research, The Thinking Materials Resource Centre, Niagara Centre, OISE, St. Catharines, Ontario, 1975.

28. Robinson, F. Classification and Logic. Monographs in Co-operative Curriculum Development, Niagara Centre, OISE, St. Catharines, Ontario, 1972.

29. Christensen, F. A Generative Rhetoric of the Paragraph. College Composition and Communication, Vol. 14, 1963, pp. 144-156.

30. Grady, M. A Conceptual Rhetoric of the Composition. College Composition and Communication, Vol. 22, 1971, pp. 348-354.

31. Paradoxically, the chief impediment to level 5 appears to lie in the area of knowledge programming. Despite the fact that the high schools have taken "content" (by which most teachers mean knowledge objectives) as their chief focus, the science of knowledge programming is not very far advanced. (Posner, G., and Strike, K. A Categorization Scheme For Principals of Programming Content, Review of Educational Research, Vol. 46, No. 4, pp. 665-690.)

32. Robinson, et al., op. cit.

33. Our project groups have from time to time administered general problem solving tests (i.e., not requiring specialized knowledge) to selected groups at different age levels, including the adult level. Growth curves over the years have not tended to be steeply inclined.

Table 1

LEVELS OF TEACHER BEHAVIOR IN IMPLEMENTING IMAGE OF THE REAL LIFE PROBLEM SOLVER

TEACHER'S CONCEPTION OF PROBLEM SOLVING	BASIS OF PROGRAMMING PROBLEM SOLVING COMPETENCIES	EXPECTATION/EVALUATION/ REMEDIAL ACTION	PROBLEM SOLVING METHODOLOGY EMPLOYED
Level 1			
: a mathematics "problem" is conceptualized as a difficult application of an algorithm, in the sense that the solution is not obvious or rehearsed (often identified with "c" exercises in the textbook). : problems are organized into "types" that follow directly from a particular level of algorithmic use.	: programming is based on progressive development of algorithmic skill. : when algorithmic performance is stabilized, problems which fit the algorithm under study are introduced as applications. : the teacher may use puzzles, and "think" exercises as supplementary materials.	: testing/marking is based primarily on proficiency of algorithmic use (speed, accuracy). : remedial teaching is aimed at overcoming computational deficiencies. : there is an expectation that many students simply lack the innate ability to solve problems; failure to solve application problems is attributed to such factors as poor reading skills (can't understand the problem), a lack of general or specific intellectual ability (poor spatial sense) or poor motivation.	: the teacher (or capable students) model problem solutions. : the teacher points out errors in the student's attempts at solution (you should have substituted a 2 for X instead of a). : little explicit articulation of strategy problem solver is using.

Level 2

: mathematics problem solving is conceived in terms of component skills, which collectively comprise a "model" for problem solving.
: each teacher has his own variant.
: the teacher may/may not have articulated this model in his own thinking.

: increasing algorithmic proficiency remains the dominant programming device.
: opportunities for strengthening other component skills seized as opportunities arise, but no specific programming of these skills.

: algorthmic proficiency predominates tests, but the teacher will include some test items which call for other skills (e.g., estimating an answer, formulating a problem question, determining what information is required to solve a problem.
: remedial teaching is still aimed primarily at overcoming algorithmic skill deficiencies, but the teacher may include special sessions directed to the improvement of other component skills.

: Modelling solutions remains the principal mode of teaching.
: In assisting students in difficulty with a problem, the teacher will provide cues/ prompts which relate to his conception of problem solving (e.g., why don't you draw a picture to show the problem elements?).

Level 3

: mathematics problem solving model (set of component skills) is articulated, and is the object of discussion and mutual accommodation among a group of teachers at different grade levels (e.g., primary, junior, or intermediate division).

: instruction programmed primarily by algorithmic proficiency.
: special units introduced to develop other components of problem solving model.
: units repeated over grades at increasing levels of sophistication but no explicit analysis of levels of component skills involved.

: test items administered to assess performance in other skill components of problem solving.
: some remedial teaching undertaken in relation to other skill components but computational skill remains the sine qua non.
: teacher expects some regular growth in all components of mathematics problem solving, but failure not perceived as critical as failure in algorithmic proficiency.

: the model is presented to students as a guide to their problem solving.
: specific regular instruction is given on other skill components, but with a general (unlevelled) focus.
: in assisting students in difficulty with a problem, teacher will refer to particular phases of the problem solving model (e.g., what steps have you accomplished, what should you be trying to do next?).

Level 4

: mathematics problem solving envisaged in terms of set of component skills, subsumed under a model, each of which can be defined in levels of increasing sophistication.

: instruction is programmed by the simultaneous development of the complete set of skills comprising the shared problem solving model.

: the teacher may still use a test programmed by increasing algorithmic sophistication as the main vehicle of progress, but attempts to ensure commensurate growth of other skills at each stage of computational growth.

: the teacher tests for growth in all problem solving components.

: failure to achieve to expected level in any components initiates remedial action.

: specific instruction given in all model components, focussed on moving students from one defined level to another.

: the teacher may use other problem solving components as vehicle for motivating acquisition of higher level of algorithmic sophistication (e.g., students able to formulate problem, envisage problem elements and write appropriate equation, but don't know how to solve it).

: teacher invites students to increase the complexity of a problem by increasing the level of sophistication of one of the model components.

: teacher consistently attempts to relate his/student's solutions to the shared model, and to articulate his strategy for using particular skills at the level required.

Level 5

: problem solving is envisaged in terms of a general set of skills (subsumed under a model), shared by teachers of different disciplines.

: interdisciplinary skill variants for each discipline are described by a mutually agreed set of levels of sophistication.

: instruction is programmed by development within each discipline of complete set of discipline-specific variants of the skills included in shared model.

: uniform levels of skill variants are developed across disciplines.

: an interdisciplinary comprehensive test of problem solving is used to define the breadth and extend growth in problem solving. (Specific discipline tests would indicate the attained level of skill development in each discipline).

: failure to perform to expected levels in all areas would initiate concern/remedial action (e.g., the student who had a highly developed ability in science inquiry but a poorly developed ability to analyze social issues to which the products of this ability would be applied).

: similarities/differences of problem solving models between disciplines are the object of analysis by students.

: instruction in component skills emphasizes commonalities of variants in related disciplines.

: problems of significance/interest to students is subjected to inter-disciplinary analysis in which discipline-specific skills and knowledge are brought to bear on the problem in an integrated sequence of problem solving episodes (e.g., the energy crisis might generate: an historical/economics analysis of causes; a geography/physics determination of sources of non-renewable energy; a mathematical projection of energy requirements and resources; and a political/decision-making analysis of possible government energy usage policies).

Level 6

: "real life" problem solving conceptualized in terms of a general set of intellectual skills (subsumed under a shared model) whose discipline-specific variants establish, fill and execute the frameworks for problematic situations in a representative array of real-life contexts.	: major programming vehicle is developmental problem solving tasks as they appear in the student's significant life contexts.	: primary evaluation is adequacy of performance in real-life tasks (doing well in discipline-oriented problems could be taken as a particular kind of real-life task).	: a substantial portion of the content of problem solving would be problem situations as perceived by students (supplemented by specific "skill battery"). : teacher would emphasize such qua'ities of real-life problems as: incomplete or biased data; the need for rapid solutions, under less than ideal conditions. : students would be taught to match intellectual effort to the perceived value of the outcome, and to analyze problem situations at whatever level of sophistication their importance warrants.